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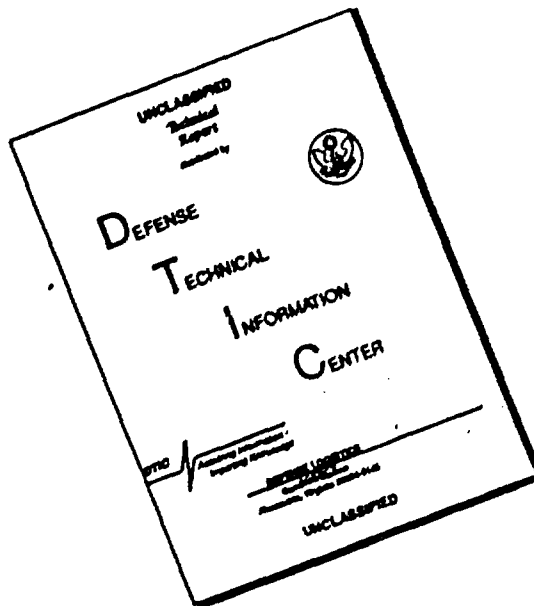
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Progress Report

# THE TANK WEAPON SYSTEM

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December, 1963

**Systems  
Research  
Group**

**Report No.  
RF-573  
PR 64-1 (U)**

**Systems Research Group  
Department of Industrial Engineering**

**The Ohio State University  
Columbus; Ohio**

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Progress Report  
1 July 1963--31 December 1963

THE TANK WEAPON SYSTEM

SYSTEMS RESEARCH GROUP  
The Ohio State University  
Engineering Experiment Station  
and  
Research Foundation

31 December 1963

Contract No. DA 15-014, AII-2965  
O. I. No. K-11326-53

U. S. Army Combat Developments  
Command Armor Agency  
Fort Knox, Kentucky

## SUMMARY

The objective of this research is to provide the information needed by the military planner in the preparation of Qualitative Materiel Requirements for armored vehicles. This report summarizes work accomplished under Contract No. DA 15-014. AII-2965 O. I. , No. K-11326-53 between 1 July and 31 December 1963.

Attention has been focused on the development of relationships to predict the mobility and firepower characteristics of future tanks with specified hardware components. Research in the areas of soft-soil agility and cross-country mobility is presented. An analysis of the effects of cant on accuracy of the tank main gun is reported, and work in target detection and first-round firing time is described. The initial formulation of a tactical performance model for tank units is explained.

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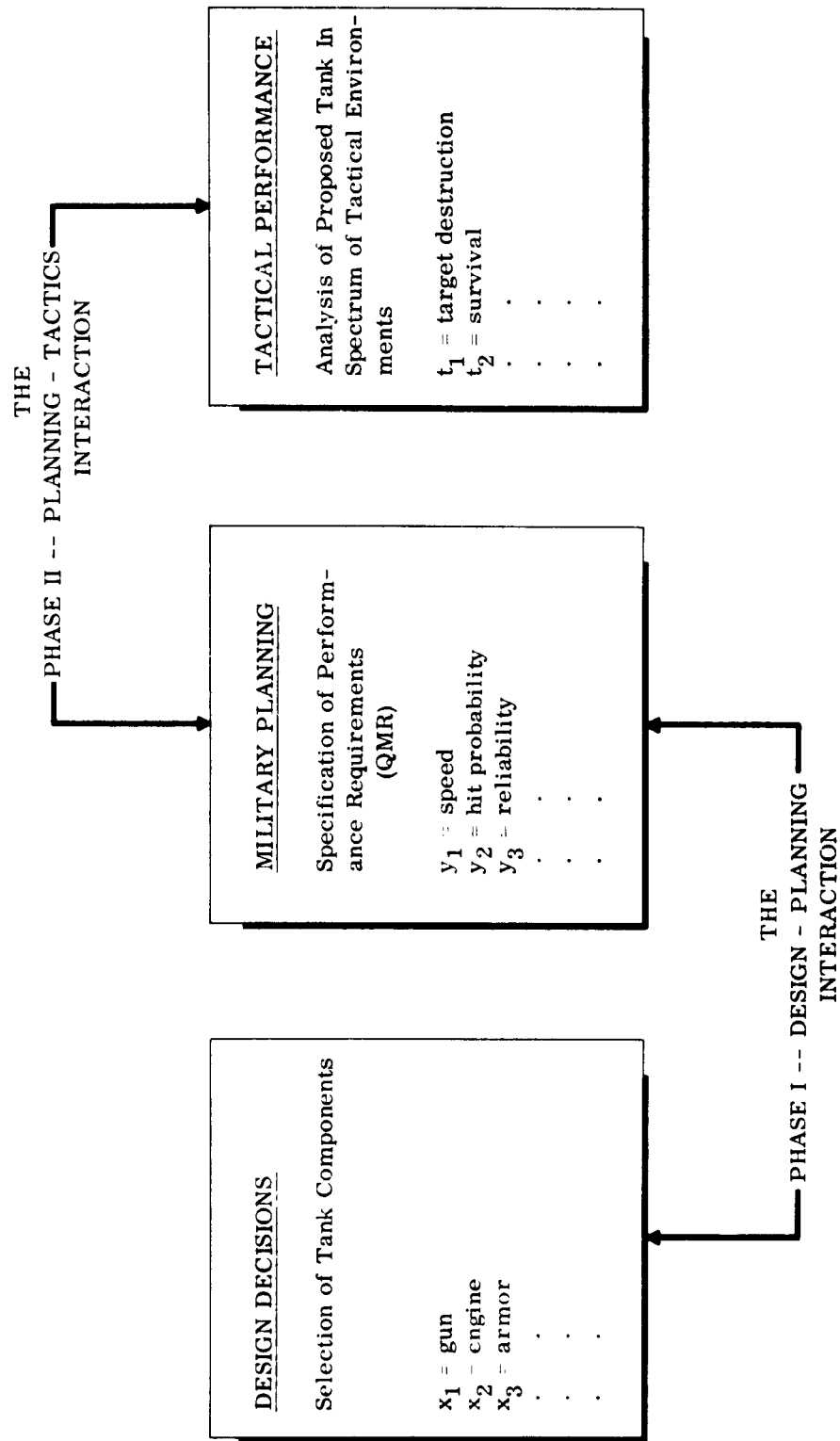


## INTRODUCTION

by  
D. Howland

The military planner preparing Qualitative Materiel Requirements stands midway between the equipment designer and user. Equipment performance characteristics are specified by the Armor Combat Developments Agency. These characteristics must meet expected tank combat requirements within man, machine and economic constraints. If the requirements do not meet both combat effectiveness and design feasibility criteria, valuable time (as well as other resources) may be lost in the development process. It is essential that the complex trade-off decisions required in the preparation of a QMR be made with both hardware feasibility and combat requirements explicitly stated (Howland, 1963). Design decisions must be objectively related to their operational consequences in the planning stage of the development process. The information required to relate design and operational decisions is shown in Figure 1.

Relationships between man and machine components and the performance of an individual tank (vehicle-component design laws) are being developed in Phase I of the project. Procedures are being developed to predict tank performance, given components of known or expected characteristics. Knowing these relationships, procedures will be developed in Phase II for relating individual tank performance to the performance of tank units in combat. The relationships developed in Phase I may be used to determine the feasibility of



OBJECTIVE: To Develop Procedure For Predicting Tactical Consequences of Design Decisions

RESEARCH OBJECTIVE

FIGURE 1

QMR specifications, and serve as the basis for predictions of combat performance.

Work in Phase I since 1 July 1963 is summarized in this report. Like the annual report of June 1963, (which should be available to the reader) it is divided into sections prepared by members of the research team. Firepower and mobility research are emphasized. The initial work on Phase II, which integrates the work in Phase I, is described. Work in the cost area has been temporarily reduced pending the acquisition of production data.

Conceptualizing the tank as a moving gun, problems of mobility and firepower are being approached as follows:

#### Mobility

The major difficulty a tank encounters in moving through soft soil, or mud, is lack of traction, rather than lack of power. Traction depends on the ability of the soil to withstand the shearing action of the tank tread. For this reason track slippage and the sinkage and inclination of the tank as the soil fails under it, are being investigated. From the design standpoint, this information is needed to specify power train and track requirements. From the operational standpoint, it must be possible to predict tank performance in soft soil, given specification of components such as the engine and track. Both aspects must be considered in preparing the QMR: the soft-soil mobility requirement specified must be adequate in combat and feasible in design.

The mobility performance characteristics of a tank traversing rough terrain are limited by the crew's ability to withstand the shocks and vibrations

to which they are subjected. Not only is top speed limited, but also crew performance will be modified in other critical aspects by accelerations due to impact loadings. In order to specify realistic cross-country speed in the QMR, the military planner must know what speeds are attainable. The general approach taken is to treat the tank as a black box which damps out accelerations resulting from cross-country mobility. The designer must have information on the expected accelerations, and the military planner must know what kinds of speeds he can realistically specify. To generate this information, accelerations are being related to speeds for different types of terrain. Shock absorber characteristics can then be specified to modify the accelerations. Given:

- (a) information on the accelerations to which the hull will be subjected, and
- (b) the way in which these accelerations may be modified by shock absorber characteristics and distribution, limits on cross-country speed may be determined as a function of human tolerance to acceleration.

### Firepower

In order to meet combat firepower requirements, targets must be detected, the round must be fired quickly, and the gunner must be able to hit what he is shooting at. These aspects of the problem are being investigated in related studies.

A number of factors have been identified as contributing to variability in hit probability (Williams, 1963). Cant, or inclination of the tank from a level position, is a controversial factor. Some experienced armor personnel

believe that it is important, while others do not. Because of conflicting opinions, this factor is being investigated. The research described in the annual report is being continued to include the effects of elevation changes due to cant.

Before a hit is possible, a target must be detected, and the round must be fired. Factors influencing detection and firing times are being analyzed. An experiment is being designed which will utilize an eye-movement camera to determine what aspects of the environment the observer looks at, and what search patterns lead to detections. Ortho-stereoscopic motion pictures will be used to simulate target backgrounds in different combat environments.

#### Combat Performance

Initial work in Phase II is in progress. A generalization of Lanchester's Laws is being studied as a means of describing and predicting the outcome of tank engagements. The information generated in Phase I will be used as inputs for this model to insure that individual tank performance (as specified in the QMR) and performance in combat will be based on obtainable man and machine performance capabilities.

SOFT SOIL MOBILITY  
by  
W. Perloff and K. Nair

Introduction

At the present time the investigation is concentrated on two major aspects of soft soil mobility which were not considered in detail in the annual report (Perloff and Nair, 1963): the effect of track slippage on mobility, and the factors which effect tank sinkage and inclination. These factors influence the expression for soft-soil mobility derived in the annual report (Perloff and Nair, 1963, p. 34).

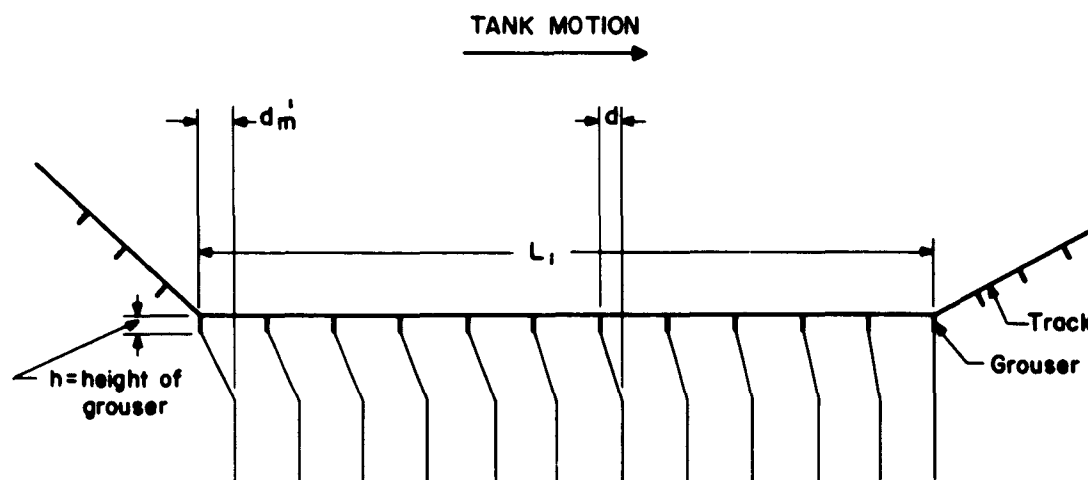
Consideration of the Effect of Tread Slippage on Mobility

When a track laying vehicle moves across a surface without slipping, the velocity of that portion of the track which is in contact with the ground surface is zero. When the velocity of the track in contact with the ground surface is not zero the track is said to be "slipping" on the soil. The effect of this slip on mobility can be divided into two parts: 1) the effect of slip on the mobilization of soil shear strength (i. e. , thrust) along the portion of the track in contact with the soil, and 2) the sinkage and change in inclination of the tank due to the removal of soil underneath the track. These are discussed in detail below.

1. In the analysis presented in the annual report, it was assumed that the full shearing strength of the soil was mobilized all along the length of the track in

contact with the soil. However, this is only a first approximation, since degree of mobilization of soil shear strength depends on the unit shear strain to which the soil is subjected. Hence, in order to determine the thrust force between the track and the soil, the distribution of shearing stress on the base of the track must be known. This distribution depends in turn upon the stress-strain characteristics on the soil, and the strain caused by the tread slippage in the soil beneath the track.

The deformation and strain in the soil caused by a slipping track are illustrated in Figure 2. This figure shows an idealized track "slipping" on the soil. The vertical lines in the soil represent planes in the soil which are initially vertical. At some depth below the track, these planes are bent due to



SOIL STRAIN AND DISTORTION CAUSED BY TRACK SLIPPAGE

FIGURE 2

the strain caused by the track slip. The soil distortion at the interface between the grousers and the soil is denoted by "d." It may be seen that "d" equals 0 at the front end of the track where it just comes in contact with the soil and increases linearly to a theoretical maximum value " $d'_m$ " at the point where the track just leaves the soil, as illustrated in Figure 3. The theoretical maximum distortion may be related to the slip as follows:

The velocity of slip may be defined as:

$$v_s = v_T - v_v \quad (1)$$

where  $v_T$  is the velocity of the tank hull relative to the portion of the track in contact with the ground.  $v_v$  is the actual velocity of the tank. Note that for zero slip,  $v_T = v_v$ , i.e., the velocity of the portion of the track in contact with the ground is zero.

The per cent slip may be defined as:

$$i = \frac{v_s}{v_T} = \frac{v_T - v_v}{v_T} = \left( 1 - \frac{v_v}{v_T} \right) \quad (2)$$

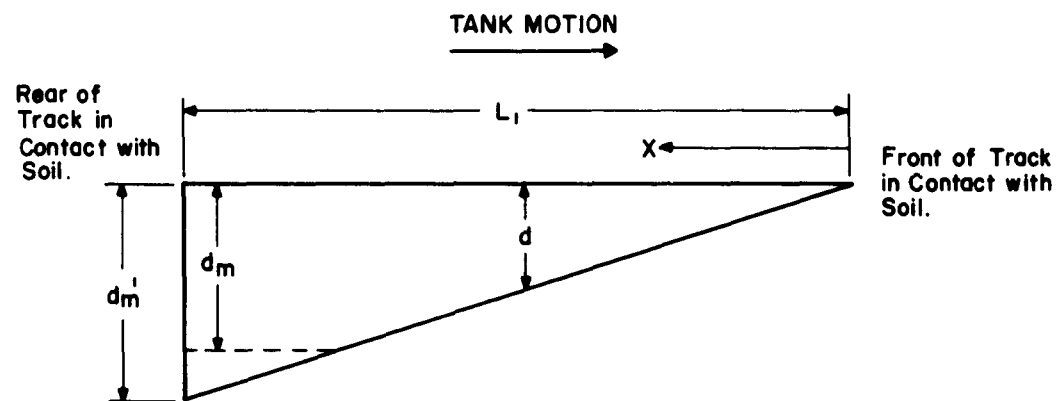
then,

$$d'_m = v_s t \quad (3)$$

where  $t$  is the time in which a given soil element is distorted the maximum amount. If  $L_1$  is the distance along which  $d'_m$  has occurred,

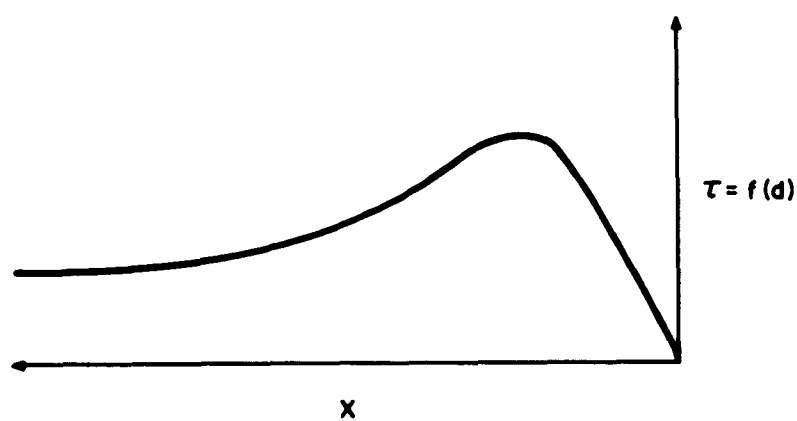
$$t = \frac{L_1}{v_T}$$





VARIATION OF SOIL DISTORTION AS A FUNCTION  
OF POSITION UNDER THE TRACK

FIGURE 3



SHEAR STRESS ON THE BASE OF THE TRACK  
AS A FUNCTION OF POSITION

FIGURE 4

and 
$$d'_m = (v_s) \frac{L_1}{v_T} = \left(1 - \frac{v_v}{v_T}\right) L_1 = L_1 i .$$

Since the distribution of distortion is linear along the base of the track (cf. Figure 3), the distortion at any point is

$$d = d'_m \frac{x}{L_1} = ix \quad (4)$$

The actual maximum soil distortion,  $d_m$ , may be less than  $d'_m$ . This difference is dictated by the stress-strain relationship of the soil, and the distortion at which the deviation occurs depends upon the strain at which the soil can no longer maintain continuity and will separate. When the distortion exceeds  $d_m$ , soil is actually removed and carried away by the track to be deposited at the rear of the tank (see Figure 3). This type of soil removal is most graphically illustrated in the case where a tank becomes firmly embedded in the soil, and its tracks are spinning. In this case, a mound of soil is quickly deposited behind the tank.

Once the soil distortion is determined at every point, it is possible to compute the shear stress distribution along the base, if the stress-strain relationship for the soil is known, and the distortion can be expressed in terms of unit strain. In general,  $\tau_x$  is a function of  $d$ , i. e.,

$$\tau_x = f(d)$$

where  $\tau_x$  is the shear stress at any point under the track, and  $d$  is a function of  $i$  and  $x$ . Such a relationship is shown in Figure 4. The thrust force per unit width of track,  $T$ , is the area under the curve in Figure 4, i. e.,

$$T = \int_0^{L_1} f(d) dx$$

In order to relate the soil distortion to the shear stress mobilized, Bekker (1956) performed plate shear tests on many soils. The results of these tests were approximated by a hyperbolic trigonometric function. The shape of the curve was fitted to the observed stress-strain curve by choosing the four constants of the equation related to soil parameters. Utilizing this expression in conjunction with the distribution of distortion at the base of the track it is then possible to obtain the shear stress distribution. This approach, while ingenious, is still subject to several difficulties and limitations.

- a. The degree to which the assumed function fits actual stress-strain curves depends on soil type and the strain being considered.
- b. The determination of the stress displacement curve for the soil requires a field test involving rather elaborate equipment. This test is essentially a shear plate test in which a plate with projections (i. e. . grousers) is subjected to normal and shearing forces at the soil surface. The parameter which determines the degree of mobilization of shear strain in the soil is the unit strain to which the soil is subjected, not the measured distortion. Thus, the determination of mobilized shear strength in the manner previously described depends upon the fact that the strains in the soil resulting from the plate shear

test will be the same as those underneath the slipping track. Since the plate used in testing is of a different width than the track, there is no reason to believe that the strains will be the same. Therefore, a direct comparison of distortions underneath the tank and underneath the plate is not possible, without considering modifications necessary due to the difference in size.

- c. Once the stress-strain, or stress-distortion curve has been determined, integration of the curve to find the total thrust must be done graphically or numerically. Since analytical integration of Bekker's function is not possible at this time, and computers are available to integrate numerically to any desired degree of accuracy, there seems to be no inherent advantage in replacing the actual stress-strain curve with mathematical equations which may deviate substantially from the actual curve.

The problem then resolves itself into the determination of unit shearing strain underneath the track, and the development of some procedure whereby the stress-unit shearing strain relationship for the soil can be determined. The laboratory tests which are commonly used by engineers to elucidate the stress-strain relationship for a soil are either the cylindrical compression test or the direct shear test. If the cylindrical compression test is properly performed, the determination of unit shearing strain is a relatively

straightforward procedure. Although the direct shear test appears to simulate the mode of shearing deformation underneath the track more closely than the cylindrical compression test, the nonuniformity in strains which occur in a direct shear specimen make determination of the unit strain exceedingly difficult.

Analysis of the unit strain in the soil underneath a tank track as related to the track slip is presently in progress. In addition, the correlation between this strain and the unit strains which occur in the cylindrical compression and direct shear test are also being studied. When these relationships have been evaluated, the total thrust can be determined by the following procedures:

- a. From the actual stress-strain curves, the stresses which correspond to the unit strains produced by the tank track can be determined.
- b. A stress-distortion curve can then be determined from these stresses.
- c. This can be numerically integrated on the digital computer to arrive at the actual value of the total thrust.

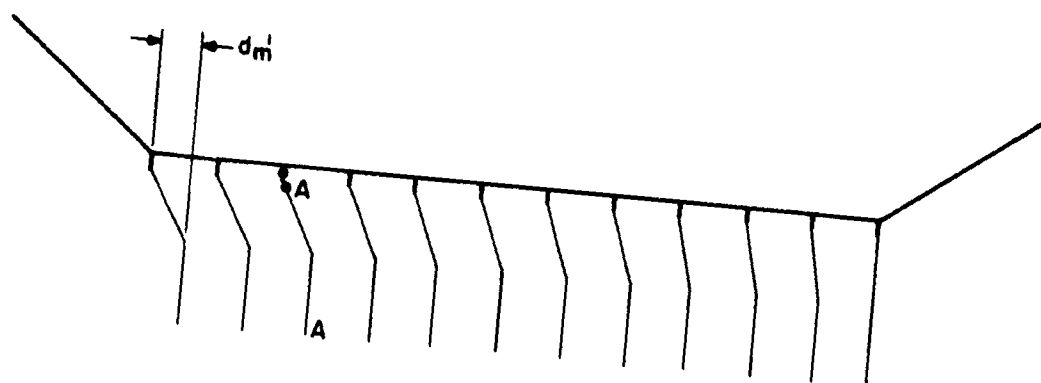
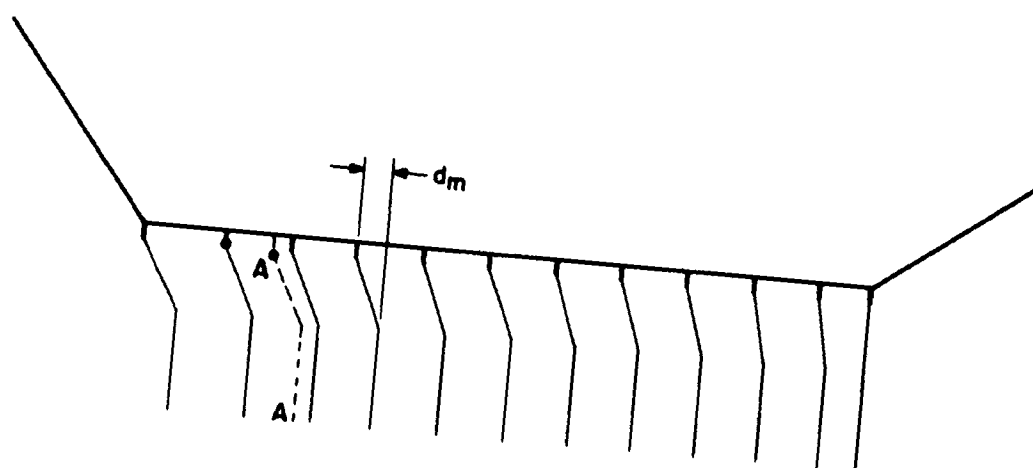
One additional complicating factor must be considered: The shearing resistance of many soils is proportional to the normal stress applied to these soils. Hence, the sinkage and change in inclination caused by slip, which are discussed below, will influence the normal pressure on the soil beneath the track, and therefore the value of the maximum shear strength. Hence, the thrust, as a function of slip, is also related to the sinkage caused by slip.

## 2. Additional Sinkage and Change in Inclination of the Tank Due to Slip.

As indicated above, the maximum distortion of the soil,  $d'_m$ , takes place at the rear of the vehicle, and is equal to  $iL_1$ . Bekker (1956, 1960) suggests that soil distortion may be identified with soil removal. Based on this assumption, the change in slope induced in the tank,  $\Delta\beta$ , can be expressed as

$$\Delta\beta = \tan^{-1} \left( \frac{2h d'_m}{L_1^2} \right) \quad (\text{Bekker, 1956})$$

where  $h$  is the grouser height. For a tank which is undergoing fifty per cent slip, this equation results in a change in inclination of less than 0.1 degrees. The indicated increase in inclination of the tank is much smaller than observation of a real tank imbedded in the soil would suggest. Figure 5 illustrates the fallacy in the preceding argument. Figure 5a represents the case where the track "slip" consists entirely of distortion of the soil. It can be seen that even though the track appears to slip, adjacent soil particles are still in contact, and there is no removal of soil material from under the track, and therefore no change in inclination. In Figure 5b the theoretical maximum distortion  $d'_m$  has exceeded the separation distortion  $d_m$  of the soil. In this case actual soil removal will occur only in that zone where  $d$  exceeds  $d_m$ . Removal of the soil only in the rear portion of the track will cause a large normal stress concentration on that portion of the track immediately in front of the zone where the soil has been removed. It is this large increase in stress underneath the rear portion of the track, combined with the removal of soil

(a)  $d_m \geq d_m'$ (b)  $d_m < d_m'$ 

INFLUENCE OF SOIL STRESS-STRAIN PROPERTIES  
ON SOIL REMOVAL BY TRACK SLIP

FIGURE 5

by slip which causes the change in tank inclination with track slippage. It is anticipated that the analysis of this approach which is underway, will lead to more realistic values for rear end sinkage and changes in inclination as a consequence of slip.

#### Load Sinkage Relationships

An extensive survey of the available literature has yielded no useful load-sinkage relationship which is valid over the large range of displacements (cf. Figure 6) observed in tank sinkage. A separate laboratory study<sup>1</sup> is in progress to determine such a relationship, at least for cohesive soils. Figure 6 shows the results of typical tests performed on smooth 2" x 2" footing and 1" x 1" square plates punching into a saturated remolded cohesive soil. It is interesting to note that the results for the 1" x 1" footing and the 2" x 2" footing are quite similar when the sinkage is plotted in terms of the measured sinkage divided by the width of the plate ( $z/b$ ). It is also interesting to compare these experimental values with the load sinkage relationship suggested by Bekker (cf. the annual report):

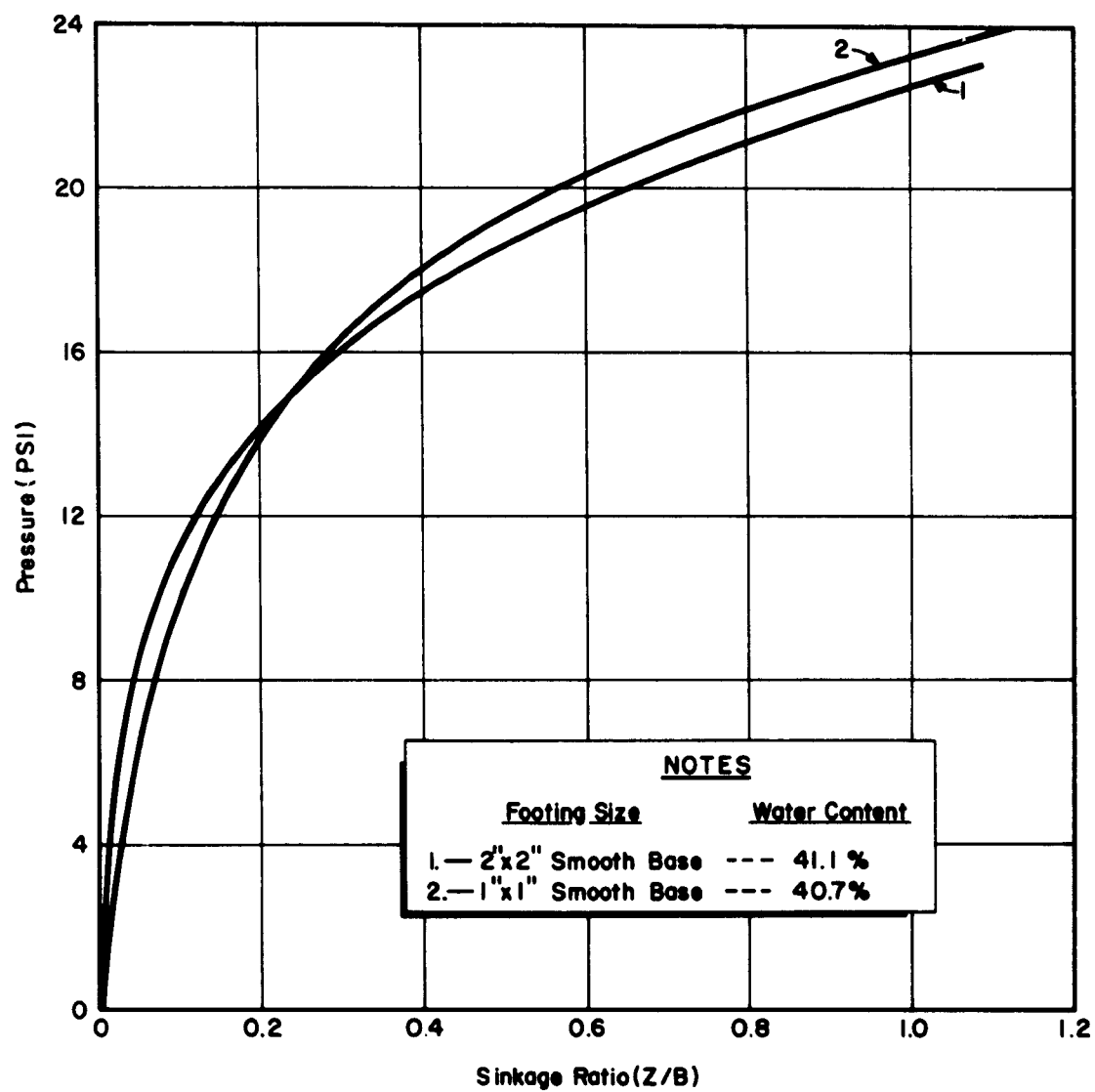
$$p = \left( \frac{K_c}{b} + K_\phi \right) z^n$$

where  $p$  is the plate pressure,  $b$  is the plate width,  $z$  is the sinkage, and

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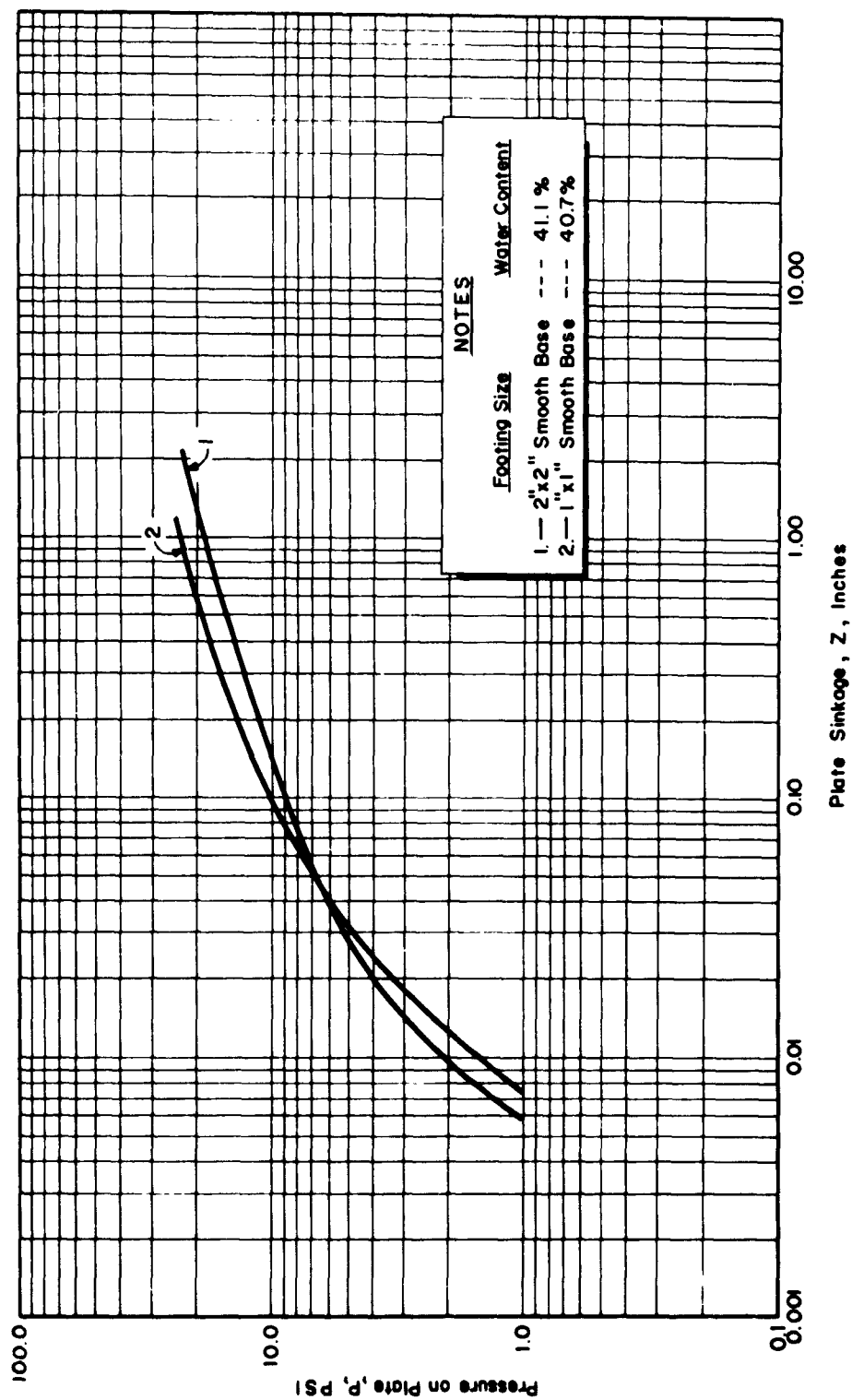
1. "A Study of Load-Sinkage Relations for Cohesive Soils," Contract No. DA-33-019-AMC-271(T) with Land Locomotion Laboratory, Army Tank Automotive Command.





PRESSURE-SINKAGE CURVES FOR MODEL PLATES

FIGURE 6



TEST PLOT OF PLATE TEST DATA FROM FIGURE 6

FIGURE 7

$K_c$ ,  $K_\phi$ ,  $n$  are constants. If the equation suggested by Bekker is a valid representation of the actual data obtained, then a logarithmic plot of test results of pressure versus sinkage would yield a straight line. The data have been plotted in this form in Figure 7 for both model tests. The deviation from the straight line is obvious. A mathematical expression for the observed load-sinkage relationship has not yet been obtained, but efforts are continuing in this direction.

## ROUGH TERRAIN MOBILITY

by  
D. R. Bussman

### Introduction

A moving tank encounters varying conditions of surface geometry and soil resilience. On smooth, paved roads, the crew can utilize the vehicle's power to approach maximum design speed. Off the road, however, the maximum speed of a tank is not governed by the available power, but rather by the vibration-tolerance of the crew. Accordingly, the average velocity, an important cross-country mobility criteria, is assumed to be dependent on:

1. Accelerations due to impact loadings which limit crew-controllability of the tank, and
2. Crew fatigue which is greatly increased by sustained exposure to low amplitude vertical and horizontal accelerations (Goldman, 1948).

This study is being conducted to determine the limiting cross-country speed as a function of the vehicle's design parameters, the terrain geometry, and the crew's tolerance to vertical, pitch, and roll accelerations.

### Method

The rough terrain mobility analysis initiated by Mahig (1963) has been modified in the following ways: A three dimensional vehicle is being analyzed which is capable of bounce (vertical), pitch, and rolling motion. The vehicle is assumed to be travelling at a constant horizontal velocity. Basically, two general types of terrain are considered; (a) discontinuous, i. e., a step function, and (b) continuous, which can be described mathematically as functions of the displacement from some reference. More than one bogie wheel can be displaced by the terrain at any time, and the left and right sets of wheels can encounter terrain of the same or different characteristics.

The vehicle is assumed to have a spring of equal stiffness at each bogie wheel. Shock absorbers are also assumed at each wheel, but their damping coefficients may be varied from wheel to wheel. This assumption allows a study of the effects of shock absorber distributions, an important parameter in suspension design.

In order to determine the motion of the vehicle, the differential equations describing bounce, pitch and roll motion of the vehicle must be derived and solved. The differential equations are obtained as follows: The vehicle is assumed to be travelling over terrain at a constant velocity. The force acting

on the body of the vehicle at any bogie wheel is expressed as a function of the spring and damping constants, and the relative displacements and relative velocities of the vehicle body with respect to the wheel. The wheel displacement and the wheel velocity are related to the terrain. Taking into account all forces acting on the vehicle, the equations of motion are written for the body of the vehicle. Using the three equations of motion, the following differential equations for bounce, pitch, and roll of the vehicle are derived (Bussman, 1963).

$$\ddot{Y} + B\dot{Y} + AY = -K_1 - K_2\theta - K_3\dot{\theta} + \sum_{i=1}^N K_{22i} Y_{wi} + \sum_{i=1}^N K_{23i} \dot{Y}_{wi} \quad (1)$$

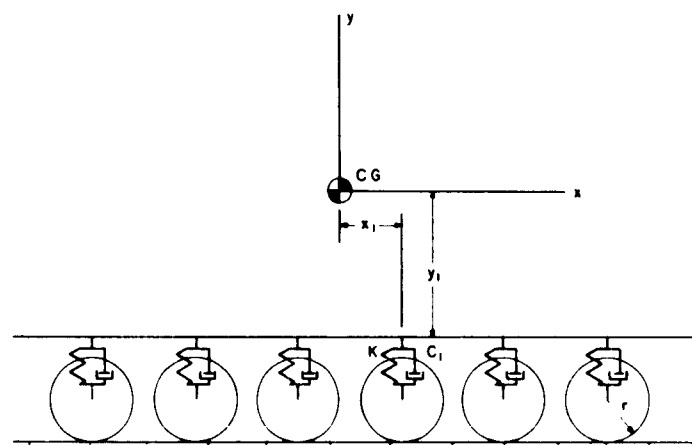
$$\ddot{\theta} + K_4\dot{\theta} + K_5\theta = -K_7 - K_6Y - K_8\dot{Y} + \sum_{i=1}^N K_{24i} Y_{wi} + \sum_{i=1}^N K_{25i} \dot{Y}_{wi} \quad (2)$$

$$\ddot{\phi} + K_{13}\dot{\phi} + K_{14}\phi = - \sum_{i=1}^N K_{26i} Y_{wi} - \sum_{i=1}^N K_{27i} \dot{Y}_{wi} \quad (3)$$

where  $Y$ ,  $\theta$ , and  $\phi$  represent bounce, pitch, and roll, respectively. The dot in the equations denotes differentiation with respect to time. Hence,  $\dot{Y}$  and  $\ddot{Y}$  are the bounce velocity and bounce acceleration, respectively.  $N$  is the number of wheels. All coefficients in the equations are constants which depend upon the spring and damping characteristics of the suspension system, the mass of the vehicle, the mass moments of inertia about the pitch and roll axes, and the position of the various wheels with respect to the mass center of the vehicle. The terms  $Y_{wi}$  and  $\dot{Y}_{wi}$  are the displacement and velocity of the  $i^{\text{th}}$  wheel.

respectively. They are functions of the terrain being traversed. Note that the terms involving  $Y_{w_i}$  and  $\dot{Y}_{w_i}$  are summed over all  $N$  wheels. This shows that each wheel contributes to the forcing functions (terrain generated force which excites vibrations) on the right hand side of the equations.

Before attempting to solve the system of differential equations for a vehicle travelling over a statistically described terrain, several deterministic terrains and/or obstacles have been analyzed. The solutions provide insight for the more difficult problem of a statistically described terrain input, such as the power spectrum. The twelve wheeled vehicle schematically depicted in Figure 8 was analyzed as it traversed: (1) a sinusoidal terrain, (2) a single, smooth obstacle, and (3) a single step-type obstacle.



SCHEMATIC DIAGRAM OF VEHICLE ANALYZED. SIDE VIEW

FIGURE 8

The appropriate relations for  $Y_{w_i}$  and  $\dot{Y}_{w_i}$  are determined for each type of terrain and the resulting differential equations are solved. The solutions for each terrain input are in a general form such that the amplitudes of motion (displacements, velocities, and accelerations) are expressed in terms of design variables, such as spring constant, damping coefficient, damping distribution, distances between the wheels, and the vehicle mass. With the amplitudes expressed in this form, it is possible to study the effects of the various design parameters on the motions of the vehicle. For details, see Bussman (1963).

(1) Sinusoidal Terrain:

Consider first a sinusoidal terrain input. The amplitude of the bounce acceleration of the vehicle is proportional to the following quantity;

$$D_1 = \left\{ \sum_{i=1}^n U_{12_i}^2 + 2 \sum_{i=1}^n \sum_{j=i+1}^n U_{12_i} U_{12_j} \cos(\beta_i - \beta_j) \right\}^{\frac{1}{2}} \quad (4)$$

where:

$$\sum_{i=1}^n U_{12_i}^2 = \frac{n \left( \frac{K}{M_v} \right)^2 + \left( \frac{\alpha}{M_v} \right)^2 \sum_{i=1}^n C_i^2}{\left( \frac{NK}{M_v} - \alpha^2 \right)^2 + \left( \frac{\sum_{i=1}^N C_i}{M_v} \right)^2 \alpha^2} \quad (5)$$

and where the  $\cos(\beta_i - \beta_j)$  term depends on the vehicle design parameters (see Equation 6). The quantities in Equation 5 are defined as follows:  $n$  is the number of wheels traversing the sinusoidal terrain.  $N$  is the total number of wheels on the vehicle ( $n = N$  if all wheels traverse the same terrain).  $K$  is

the spring constant.  $M_v$  is the vehicle mass.  $\alpha$  is a terrain characteristic. and  $C_i$  is the damping coefficient for the  $i^{\text{th}}$  wheel.

Equation 4 provides an answer to the following question: If the total damping coefficient,  $\sum_{i=1}^N C_i$ , for all shock absorbers is to be some specified value  $H$ , will bounce accelerations be less with a few shocks, say four, of magnitude  $H/4$  or twelve shocks each of magnitude  $H/12$ ? This can be answered by investigating the term  $\sum_{i=1}^n U_{12_i}^2$ . For a given value of  $H$ , the denominator of  $\sum_{i=1}^n U_{12_i}^2$  does not vary; however, the numerator varies considerably as a function of the distribution of the damping among the wheels.

As an example, consider a vehicle with twelve wheels. ( $N = 12$ ). If equal dampers, with coefficients of  $H/4$ , are placed on the outer wheels only, then  $\sum_{i=1}^N C_i^2 = H^2/4$ . As an alternative, assume all twelve wheels have equal dampers of magnitude  $H/12$ . For this case,  $\sum_{i=1}^N C_i^2 = H^2/12$ , or one-third of the previous value. The total effect on the magnitude of  $\sum_{i=1}^n U_{12_i}^2$  will be determined by the relative magnitudes of the two terms in the numerator; however, the analysis suggests that it might be advantageous to use shocks at all the wheels if bounce accelerations are to be reduced.

The second term in Equation 4 can be minimized by proper selection of design parameters. This is shown by consideration of the  $\cos(\beta_i - \beta_j)$  term.

$$\cos(\beta_i - \beta_j) = \frac{1}{\sqrt{U_{12_i}^2 U_{12_j}^2}} \left\{ \left[ u_{2_j} u_{2_i} + u_{1_j} u_{1_i} \right] \cos \left[ \lambda (x_i - x_j) \right] - \left[ u_{1_j} u_{2_i} - u_{2_j} u_{1_i} \right] \sin \left[ \lambda (x_i - x_j) \right] \right\} \quad (6)$$



where  $(x_i - x_j)$  is the wheel spacing and  $U_{12_i}$ ,  $u_{1_i}$ , and  $u_{2_i}$  depend on the spring and damping constants at the individual wheels, the vehicle mass, and the terrain frequency. If the suspension components at all wheels are identical, the coefficient of the  $\sin \left[ \lambda(x_i - x_j) \right]$  term is zero. The other term inside the bracket can be minimized by proper selection of the wheel spacing.

The above discussion applies to the amplitude of the bounce acceleration for a vehicle traversing a sinusoidally described terrain. Analyses described in Bussman (1963) for the pitch and roll acceleration results in similar conclusions for this terrain. That is, in order to minimize the angular acceleration, the damping should be distributed equally among all wheels.

#### (2) Single. Smooth Obstacle:

Consider next a vehicle traversing a single, smooth obstacle described mathematically by:

$$y_o = A(1 - \cos \lambda x), \quad 0 \leq x \leq \frac{2\pi}{\lambda}, \quad (7)$$

where  $y_o$  is the terrain displacement,  $A$  is half the obstacle amplitude,  $\lambda$  is a terrain characteristic, and  $x$  is the horizontal distance along the obstacle measured from a reference axis. The amplitudes of motion (displacement, velocity, accelerations) are expressed as the sum of  $n$  terms--the number of wheels hitting the bump. In order to minimize the amplitudes, the analysis indicates that the damping should be distributed equally among all the wheels.

#### (3) Single Step-Type Obstacle:

The effect of the vehicle hitting a step-type bump was considered. The amplitudes of the accelerations are expressed as the sum of  $n$  impulsive

motions due to the  $n$  wheels hitting the bump. An analysis of the amplitudes suggests that bounce, pitch, and roll accelerations could be minimized by making the wheel radius and the number of wheels as large as possible. The damping should again be distributed equally among all wheels.

The terrains considered to date are deterministic, and accordingly, not descriptive of actual rough-terrain conditions. The next phase of the mobility study will be concerned with developing a method for determining the limiting velocity of a specified vehicle travelling over a statistically described terrain.

#### FIRST ROUND HIT PROBABILITY

- The Effects of Cant -

by

R. Williams

#### Introduction

There are differences of opinion among experienced armor personnel regarding the effects of cant on the accuracy of the tank main gun. The writer's preliminary estimates of the miss distances attributable to cant indicate that cant may well be a significant factor in first round hit probability. This view seems to be supported by the fact that devices have been built and tested to correct cant although, admittedly, with inconclusive results (Eckles, et.al., 1963). Others, however, consider the effects of cant to be insignificant.

This investigation is an attempt to isolate and quantify analytically the effects of cant in terms of horizontal and vertical miss distances. For this reason, other important factors affecting hit probability such as drift, jump and

human errors are omitted in this analysis.

### Background<sup>1</sup>

If a tank were sitting on level ground with its sight centered precisely on target at a range different from the zero range, the theoretical point of impact of its projectile would miss the aiming point in both the horizontal and vertical direction. These miss distances, respectively HMD and VMD, are due, in part, to the fact that the line of sight of the sight and the line of flight of the projectile are not coincidental. For this reason, HMD and VMD are called parallax errors.

Suppose the tank with its assumed sighting posture, were canted (i.e., rotated about its longitudinal axis). Some major effects of this action are evident.

1. The azimuth of the main gun with respect to a true vertical plane changes.
2. The superelevation of the main gun with respect to a true horizontal plane changes.
3. Sight offset from the tube changes in both the horizontal and vertical planes (parallax changes).

Specifically, the sight position (S), the trunnion position (T), the muzzle position (G), and the zero point (R) have changed with respect to the set of reference

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<sup>1</sup>Notation used in this section is explained on page 34.

axes whose origin is located at the center of the tank's turret ring. These axes lie in the true horizontal and vertical planes. Operationally, when a tank has been canted (say  $\theta$  degrees), the turret must be rotated ( $\phi$  degrees), and the elevation must be changed ( $\gamma$  degrees) in order to re-center the sight on the target. These actions, of course, would change the position of the gun tube, thus affecting the theoretical projectile impact point and, again, HMD and VMD. The annual report (Williams, 1963) contains a derivation for HMD and VMD as functions of cant and turret rotation but not elevation. It is the purpose of this section to include the effects of the required elevation changes.

#### Method

One might naturally think of the sequence of events described above as follows:

1. Sight centered on target with tank level.
2. Tank canted  $\theta$  degrees.
3. Turret rotated  $\phi$  degrees to center sight on target in azimuth.

(In practice, this may require several angular turret movements.

However, we may take  $\phi$  degrees to be the algebraic sum of these angular adjustments.)

4. Elevation changed  $\gamma$  degrees to center the sight on target in elevation. (Again,  $\gamma$  degrees may be the algebraic sum of several adjustments.)

Conceptually, however, this is no different from the sequence of statements 1.

4. 2. 3. even though such a sequence would be impossible to follow in a physical sense. With this in mind, it remains only to substitute for  $G_o$  and  $R_o$  in the HMD and VMD equations previously developed. (since  $S_o$  and  $T_o$  are unchanged by tube elevations). The expressions derived below give the positions of G and R after an elevation change of  $\gamma$  degrees and are denoted  $G'_o$  and  $R'_o$  respectively. Figure 9 shows the geometric relationship between the trunnion position ( $T_o$ ) and the muzzle positions before ( $G_o$ ) and after ( $G'_o$ ) an elevation change of  $\gamma_{TG}$  degrees of the tube.

From Figure 9, we may derive the x, y, and z components  $G'_{x_o}$ ,  $G'_{y_o}$ ,  $G'_{z_o}$  of the muzzle.

$$G'_{x_o} = T_{x_o} + (G_{x_o} - T_{x_o}) \cos \gamma_{TG} - (G_{z_o} - T_{z_o}) \sin \gamma_{TG}$$

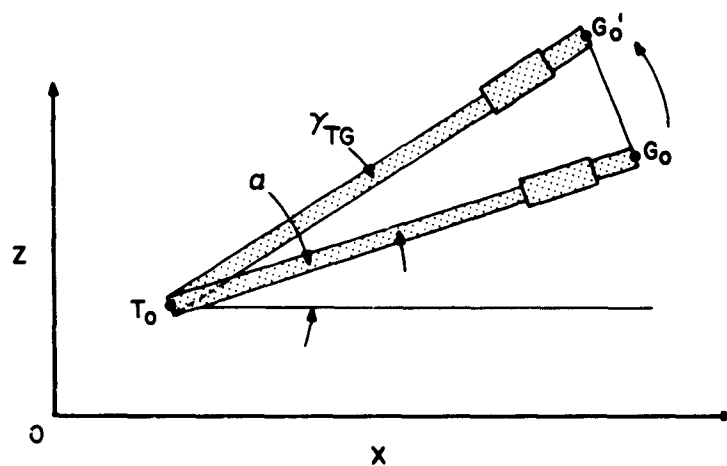
$$G'_{y_o} = G_{y_o} \quad (\text{unchanged by elevation change before cant}) \quad (1)$$

$$G'_{z_o} = T_{z_o} + (G_{z_o} - T_{z_o}) \cos \gamma_{TG} + (G_{x_o} - T_{x_o}) \sin \gamma_{TG}$$

$$T'_{x_o} = T_{x_o}, \quad T'_{y_o} = T_{y_o}, \quad T'_{z_o} = T_{z_o} \quad (\text{unchanged by elevation of tube})$$

Figure 10 shows the relationship between the sight position and the zero point (original intersection of the projectile trajectory and the line of sight of the sighting device).

The analysis of line-of-sight changes is more complex than that of the tube centerline, but is still straightforward. The positions of the zero point and sighting device after an elevation of the line of sight of  $\gamma_{SR}$  degrees are:



$T_0$  and  $G_0$ , respectively, are the trunnion position and muzzle position when the tank is level and sight is centered on target.

$\alpha$  = elevation of the tube when the tank is level and the sight is centered on target.

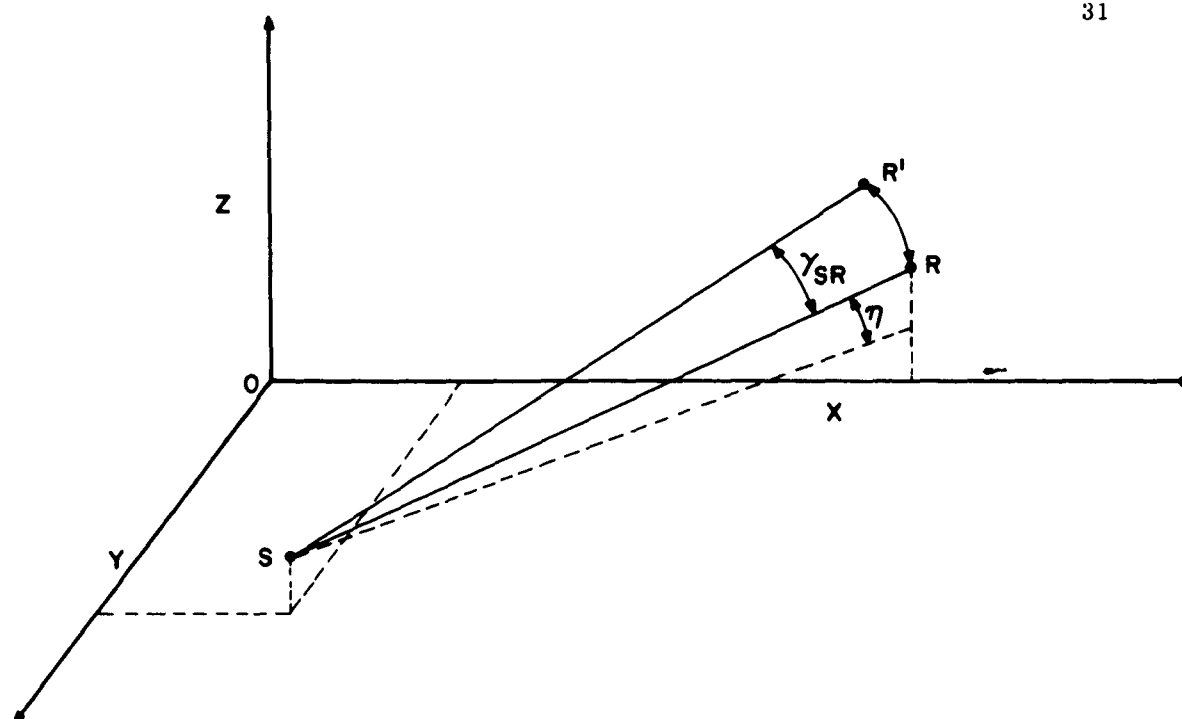
$\gamma_{TG}$  = change in elevation of the tube (in conjunction with turret rotation of  $\phi$  degrees) necessary to bring sight on target.  $\gamma$  is positive for an increase in superelevation.

Coordinate axes pass through the center of the turret ring and correspond to true horizontal and vertical planes.

POSITION OF TRUNNION AND MUZZLE BEFORE AND AFTER

ELEVATION CHANGE

FIGURE 9



S - position of the sighting device

R - original position of the zero point

R' - position of zero point after change in elevation of  $\gamma_{SR}$  degrees (no longer a "true" zero point)

$\eta$  - angle between original line of sight (S to R) and the true horizontal

$\gamma_{SR}$  - change in elevation of line of sight (corresponding to change of elevation of tube)

Coordinate axes pass through the center of the turret ring and correspond to true horizontal and vertical planes.

POSITION OF SIGHT AND ZERO POINT BEFORE AND AFTER

ELEVATION CHANGE

FIGURE 10

$$R'_{x_o} = S_{x_o} + (R_{x_o} - S_{x_o})$$

$$R'_{y_o} = S_{y_o} \left\{ 1 - \frac{\left( \sqrt{S_{y_o}^2 + (R_{x_o} - S_{x_o})^2} \sin \gamma_{SR} + (R_{z_o} - S_{z_o}) \cos \gamma_{SR} \right)^2 - (R_{z_o} - S_{z_o})^2}{S_{y_o}^2 + (R_{x_o} - S_{x_o})^2} \right\}^{\frac{1}{2}}$$

$$\left[ \frac{S_{y_o}^2 + (R_{x_o} - S_{x_o})^2 + (R_{z_o} - S_{z_o})^2 - \left( \sqrt{S_{y_o}^2 + (R_{x_o} - S_{x_o})^2} \sin \gamma_{SR} + (R_{z_o} - S_{z_o}) \cos \gamma_{SR} \right)^2}{S_{y_o}^2 + (R_{x_o} - S_{x_o})^2} \right]^{\frac{1}{2}}$$

$$R'_{z_o} = S_{z_o} + \sqrt{S_{y_o}^2 + (R_{x_o} - S_{x_o})^2} \sin \gamma_{SR} + (R_{z_o} - S_{z_o}) \cos \gamma_{SR} \quad (2)$$

$$S'_{x_o} = S_{x_o}, \quad S'_{y_o} = S_{y_o}, \quad S'_{z_o} = S_{z_o}$$

The substitution of  $G'_{x_o}$ ,  $G'_{y_o}$ ,  $G'_{z_o}$ ,  $R'_{x_o}$ ,  $R'_{y_o}$ ,  $R'_{z_o}$  respectively for  $G_{x_o}$ ,  $G_{y_o}$ ,  $G_{z_o}$ ,  $R_{x_o}$ ,  $R_{y_o}$ ,  $R_{z_o}$  in the HMD and VMD equations given in the annual report (pages 81-86) will complete the geometric representation of the effects of cant on horizontal and vertical miss distances.

The hit probability work to date has been primarily concerned with the



effects of cant on the accuracy of fire of the tank main gun. The next step in the cant analysis will be to determine the numerical magnitude of errors due to cant, and the effect of cant as a fixed bias and as a variable bias to first round hit probability. In addition, effort will be directed to revise estimates of tank parameter values. Procedures for predicting values of these parameters for future tanks will be investigated within the general framework of Brodtkin's (1957) hit probability model.

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## SYMBOLS

HMD = horizontal miss distance\*

VMD = vertical miss distance

$\theta$  = angle of cant; positive counterclockwise and negative clockwise looking forward from inside the tank

$\phi$  = angle turret is rotated to bring sight on target; positive counterclockwise and negative clockwise looking down on the tank

T = the position of the centerline of the tube at the trunnions

$T_{x_o}, T_{y_o}, T_{z_o}$  = distances from the origin to T in the x, y, and z directions,  $\theta = 0, \phi = 0$

G = the position of the centerline of the tube at the muzzle

$G_{x_o}, G_{y_o}, G_{z_o}$  = distances from the origin to G in the x, y, and z directions,  $\theta = 0, \phi = 0$

S = the position of the center of the sight

$S_{x_o}, S_{y_o}, S_{z_o}$  = distances from the origin to S in the x, y, and z directions,  $\theta = 0, \phi = 0$

R = the position of the point at which the weapon is zeroed

$R_{x_o}, R_{y_o}, R_{z_o}$  = distances from the origin to R in the x, y, and z directions,  $\theta = 0, \phi = 0$

D = line of sight range to the target

$\alpha$  = azimuth of target measured from the x axis in a horizontal plane; positive counterclockwise and negative clockwise looking down on the tank

$\beta$  = elevation of target measured from the x axis in a vertical plane; positive up, negative down

g = acceleration of gravity in ft. per second per second

$\gamma$  = the change in superelevation.

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\*All distances are measured in feet.

## TARGET DETECTION

by  
S. Stollmack

Introduction

Target detection is a critical aspect of weapon system performance. Detection capability may be measured by the expected time to detect a target in various environments, given that one is present.

Initially, the detection of stationery targets against a homogeneous background by stationery observers was studied. The intent was to extend the analysis to include relative motion (between target and observer) and nonhomogeneous backgrounds. The model proposed (Stollmack, 1963A) used empirically based detection lobe equations developed by Lamar (1959). These equations were based on measured contrast thresholds for homogeneous backgrounds (Stollmack, 1963B) and involved detection distances far in excess of tank firing capabilities. The model could not be revised without experimentally measuring contrast thresholds for backgrounds that were more representative of tank combat environments. Designing such an experiment meant controlling the characteristics of complex backgrounds that affected contrast thresholds. These characteristics are not explicitly known, although it has been conjectured that shape and size relationships and contrast distributions are important. In addition, too little was known about the validity of the following assumptions made in the original model:

1. Search is composed of many discrete eye fixations (known

as glimpses). The time between glimpses is constant.

2. Glimpse direction is a random variable independent of time.  
i.e., the direction of fixation is independent of the direction of previous fixations.
3. Backgrounds are homogeneous. i.e., target-to-background contrast is not dependent on target's position. and
4. As a result of 2 and 3 above, the probability of detecting a target at any one glimpse is constant for all glimpses.

Questions about the validity of these assumptions led to a restatement of the research direction (Stollmack, 1963B).

### Background

The correspondence between experimental and real environmental conditions must be established to validate a predictive detection model. Most experiments in target detection have dealt with abstract displays. Complex backgrounds, for example, have been simulated with nonsense forms (Boynton & Bush, 1957). Since not enough is known about the effects of background complexity to design effective abstract displays, information concerning the detection process should be obtained from actual field experiments.

A study of the detection process requires knowledge of what characteristics in the actual environment the observer appears to utilize in searching for targets. We assume that inferences regarding the influence of environmental characteristics on the detection process can be drawn from a study of

the relationship between the direction of an observer's initial and successive eye fixations. More concisely, the observer's search pattern should be investigated in connection with the following:<sup>1</sup>

1. What features of the environment determine where an observer will look ?
2. How long will the eyes fixate in any given area while searching?  
Is this time dependent on characteristics of the background?
3. Is the fixation time constant during search?
4. What is the relation between target detection and search pattern?  
(It is possible to infer a relation by investigating the search patterns of successful and unsuccessful observers.)
5. Are search patterns generally the same from subject to subject or for any one subject from scene to scene?
6. Is the fixation rate dependent on the environment? Does it differ markedly from subject to subject for any one environment?

#### Method

In order to reduce the variability in experimental conditions (lighting, visibility, etc.) the environment will be simulated with ortho-stereoscopic motion pictures of actual terrain. Backgrounds, representative of combat situations, will be photographed for various target locations and coded as to

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<sup>1</sup>A method of investigating search patterns will be explained later in this paper.

contrast. cover. distance and visibility. These motion pictures will be shown to subjects who will be instructed to search for targets. Eye movements will be recorded and superimposed on the visual scenes by a head-mounted eye camera similar to that developed by Mackworth (1960). The time to detect will be the dependent variable. Variables subject to experimental control will include:

1. Variables within the scene;
  - a. illumination (daylight conditions)
  - b. target distance
  - c. target location
  - d. inherent contrast
  - e. background complexity
  - f. target motion
2. Contrast and illumination, i. e., target-to-background contrast and illumination of the scene can be altered by film processing;
3. Observer movement relative to scene, i. e., motion can be imparted on the observer or the recording camera;
4. Search time;
5. Visual device used by the observer;
6. Vibration of observer.

Subjects will be run with and without visual devices. Without devices. for each subject and scene. average glimpse time (g). detection time (T). the number of glimpses used to detect (NF). and the distribution of glimpses will be measured. With visual devices. the time per device fixation ( $\tau$ ). and. if possible. the number of glimpses per device fixation will be measured. These data may be used to determine the following relationships.

Consider an environment  $\theta_i$  described by the particular scene, target-to-background contrast (C), meteorological visibility (V), target range (R), etc. It is hypothesized that  $\overline{NF}_i$ --the average number of glimpses needed to detect in environment (i) without use of a vision device--is dependent on the measurable environmental factors noted above. The relation between  $\overline{NF}_i$  and the environmental factors is then empirically determined:

$$\overline{NF}_i = f(C, R, V, \dots) \quad (1)$$

For a number of presentations of environment (i), the probability distribution function of detecting on or before the  $N^{\text{th}}$  glimpse in environment (i)--  $P_i(N)$ -- is determined from the data:

$$P_i(N) = \frac{\text{Number of presentations of environment (i) where detection was made within } N \text{ glimpses}}{\text{Number of presentations of environment (i)}} \quad (2)$$

For a set of visual devices  $D = (d_1, \dots, d_n)$ , the following constant can be determined:

$$K_{ij} = \frac{\overline{N}_{ij}}{\overline{NF}_i} \quad \begin{array}{l} i = 1 \dots n \\ j = 1 \dots m \end{array} \quad (3)$$

where:

$\overline{N}_{ij}$  = the measured average number of visual device fixations used to detect with environment (i) and device (j).

$K_{ij}$  = a constant for environment (i) and device (j).

$K_{ij}$  is then related to characteristics of the visual device such as magnification (M), aperture size (A), etc.:

$$K_{ij} = g(M, A, \dots) \quad (4)$$

To predict detection time ( $T_i$ ) for a proposed tank operating in environment (i) without a visual device;

$$T_i = \overline{NF}_i (g_i) \quad (5)$$

where  $g_i$  is the glimpse time in environment (i) and  $\overline{NF}_i$  is determined from equation (1). To predict detection time of a proposed tank operating in environment (i) with a visual device, the following procedure is used: The expected number of device fixations to detect-- $E_{ij}(N)$ --is calculated from equations (1) and (4);

$$E_{ij}(N) = K_{ij} \overline{NF}_i \quad (6)$$

then;

$$T_{ij} = E_{ij}(N) \cdot \tau_{ij} \quad (7)$$

where:

$T_{ij}$  - expected time to detect with environment (i) and device (j)

$\tau_{ij}$  - time between device fixations for environment (i) and device (j).

These procedures will be used to determine the important aspects of complex backgrounds that effect detection time. Once the aspects of complex backgrounds that effect detection time are known, it will be possible to describe combat environments in terms of variables affecting target detectability and functionally relate them to predict detection performance.



FIRST ROUND TIME  
by  
E. C. Sambuco and D. Heuser

Introduction

After World War II it was noted that, in tank to tank combat, the tank firing the first round was usually victorious. This was supposedly due not only to the fact that their probability of hitting first was greater, but also due to the "shock" effect on the opposing crew. It is, therefore, desirable to reduce the time required to perform this activity in the course of battle. Before this can be accomplished, however, a means of relating the factors comprising this time must be developed. The time period from the commander's acquisition of a target to the firing of the first round will be referred to as first round time ( $T_1$ ). The selection of relevant variables and possible approaches to developing a model to predict  $T_1$  are discussed in this section.

Method

Two approaches to constructing a model to predict first round firing time are possible. The first is an operational approach which would utilize data collected by the Ballistics Research Laboratory (Hardison, Killian, Wolfe, Fieldman, 1955). The data consist of times to load and times to lay the weapon for combinations of tank fire control systems and visual devices. Empirical relationships can be derived relating load time to the hardware characteristics of the system (volume, projectile size, etc.) and lay time to the performance characteristics of the visual devices utilized. Although this

approach would provide some measure of predictability. it confounds many of the activities comprising the firing sequence.

An alternative approach would consider, in detail, the various activities of the firing sequence. The following activities have been selected from the Tank Gunnery Manual (FM 17-12, 1961) as basic variables relevant to the firing sequence:

- A. Time to alert crew and prepare for the firing instructions (ALT).
- B. Time to stop tank (STOT).
- C. Time to determine range of target (RANGT).
- D. Time to swing gun roughly on target (SWIT).
- E. Time to make final lay (LAYT).
- F. Time to select ammunition and load gun (LOAT).
- G. Time to feed information to computer (COMT).
- H. Extra increments of time or delays (EXT).

These basic variables are dependent on many other factors:

A. ALT

- 1. A factor related to the training of the crew (TRAINF).
- 2. A factor related to the configuration of equipment within the tank with respect to the crew (HUMF1).

B. STOT

- 1. Velocity of tank upon sighting target (V).
- 2. The deceleration capability of the tank (DECEL).
- 3. Environmental factors such as terrain and soil type (ENVIR1).
- 4. Range of target (R).
- 5. Human factors relating to the use of the range finder; such things as ease of adjustment, head rest, and configuration of controls (HUMF2).

6. Size of target (S).

#### C. RANGT

1. The method of ranging used (METHR).
2. Environmental factors such as weather conditions and target background (ENVIR2).
3. Optical properties of range finder if any (OPTPR).
4. Range of target (R).
5. Human factors relating to the use of the range finder. such things as ease of adjustment. head rest and configuration of controls (HUMF2).
6. Size of target (S).

#### D. SWIT

1. Degrees turret must be swung (DTS).
2. Rate of turret traverse (RTT).

#### E. LAYT

1. Weather conditions and target background.
2. Target range.
3. Optical properties of the periscope used (OPTPP).
4. Human factors relating to the use of the periscope: such things as ease of adjustment. head rest. and configuration of controls (HUMF3)
5. Size of target (S).

#### F. LOAT

1. Type of loader. automatic or manual (TL).
2. Position of shell when the firing procedure starts. in gun. ready rack. or storage (POS).
3. Human factors relating the weight and size of the shell. the ease with which it can be loaded. and the configuration of equipment (HUMF4).
4. The time to turn the safety off (SAT).

#### G. COMT

1. Type of computer used (CT).
2. Human factors relating the location of the computer and the crew. the number and type of controls to be manipulated (HUMF5).

## H. EXT

1. Communications lags (CL).
2. Delays (D).
3. Time to push trigger (TTT).
4. Other (O).

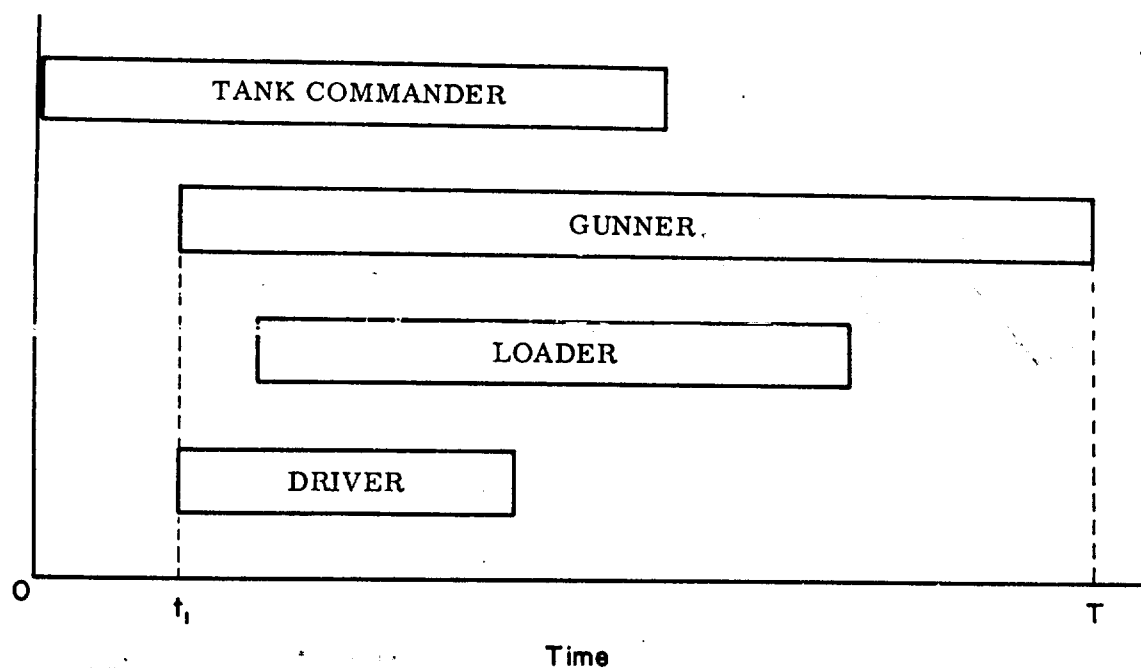
These activity times must then be functionally related to the relevant hardware, training, and environmental factors as shown qualitatively below.

- A. ALT      $f_1$  (TRAINF. HUMF1)
- B. STOT      $f_2$  (V. DECEL. ENVIR1)
- C. RANGT      $f_3$  (METHR. ENVIR2. OPTPR. R. HUMF2. S)
- D. SWIT      $f_4$  (DTS. RTT)
- E. LAYT      $f_5$  (ENVIR2. R. S. OPTPP. HUMF3)
- F. LOAT      $f_6$  (TL. POS. HUMF5. SAT)
- G. COMT      $f_7$  (CT. HUMF5)
- H. EXT      $f_8$  (CL. D. TTT. O)

Finally, the seven factors will be used to predict first round time.

$$\text{FIRST ROUND TIME} = F(\text{ALT. STOT. RANGT. SWIT.} \\ \text{LAYT. LOAT. COMT. EXT}) \quad (1)$$

A major problem in developing equation (1) above, is the overlap in time between the basic activities. Figure 11 depicts the sequence of activities performed by crew members and indicates the overlap. A precedence diagram (Moore, 1962) was developed to facilitate a more definite representation of



Time O - Target Detected

Time T - 1st Round Fired

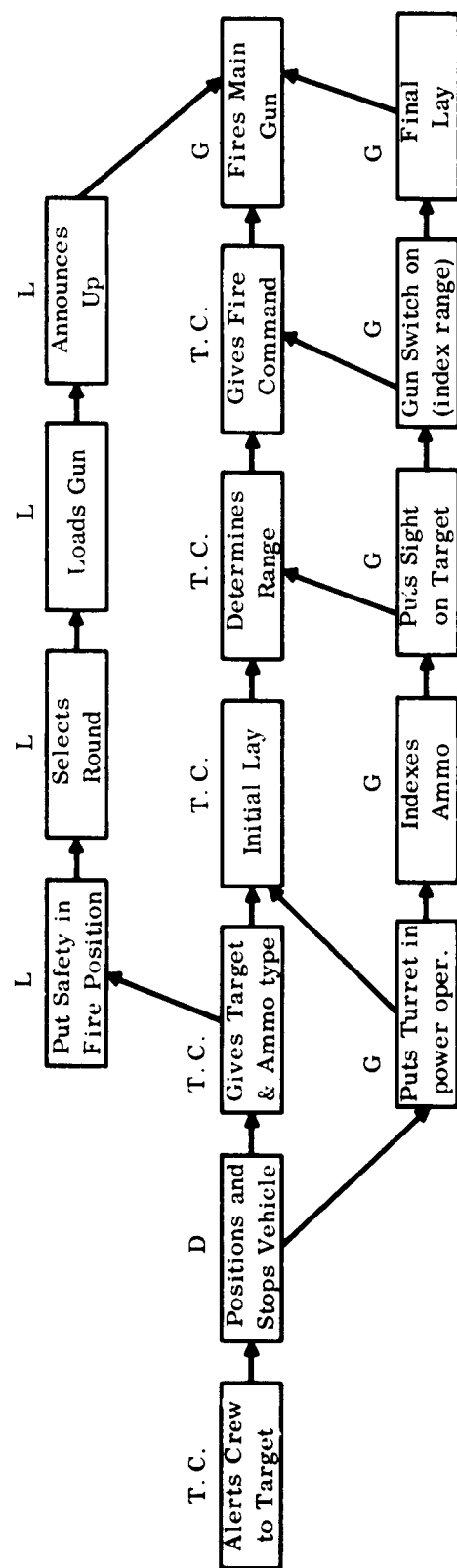
Time  $t_1$  - Communication Time

#### SEQUENCE OF CREW ACTIVITIES WITH OVERLAP

FIGURE 11

activity overlap (Figure 12). The diagram shows the detailed sequence of activities, and indicates which events must precede others.

Although still in its formative stages, the qualitative formulation presented appears promising if data for the various activities can be obtained. It appears reasonable to assume that activity-time data could be generated using tank simulators in the Armor School.



T. C. - Tank Commander  
 D - Driver  
 L - Loader  
 G - Gunner

PRECEDENCE DIAGRAM FOR A TYPICAL FIRING PROCEDURE

Figure 12

## PHASE II -- COMBAT EFFECTIVENESS

by  
Seth Bonder

### Introduction

The first phase of the project is concerned with the development of relationships and procedures to predict the performance capabilities (Military Characteristics -- see Figure 13) of next generation tank systems. The military planner specifies the component hardware to be used (engine, main gun, etc.) and the environment in which the system is to operate (Western Germany, Korea, etc.). The relationships will predict, for each system produced, its agility and speed characteristics, the accuracy, timeliness, and lethality of



Speed	= _____	Time to Detect Targets	= _____
Acceleration	= _____	Time to Fire First Round	= _____
Cruising Range	= _____	First Round Hit Probability	= _____
Fording Depth	= _____	Time to Fire Succeeding Rounds	= _____
Ballistic Protection	= _____	Succeeding Round Hit Probability	= _____
Reliability	= _____	Projectile Effectiveness	= _____
Combat Load = _____			

### MILITARY CHARACTERISTICS

Main Battle Tank M-X

FIGURE 13

its weapon, its target acquisition capabilities, etc., for the particular environment specified. In the second phase of the study (Howland, Bonder, 1963, p. 162), this work will be integrated in a model to predict the combat effectiveness of tactical units equipped with the proposed tank. The purpose of this section is to briefly present the exploratory efforts in the development of the model.

Since the responsibility of the tank system planner is the development of equipment and tactics for armored units of the future, the method of measuring operational performance should be sufficiently general to facilitate analyzing a broad spectrum of operational situations (attack, defend, etc.). The model should permit prediction of proposed tank system combat performance when employed in different environments, using different tactics, and against a range of enemy weapons. Prediction of combat performance must be based on attainable capabilities of each tank in the unit. Accordingly, the predicted performance capabilities of each tank (some of which are shown in Figure 13) will be used as inputs to the combat effectiveness model. This will facilitate an objective determination of the military importance of mobility, firepower, and protection factors in different operational situations.

#### Method

One of the earliest attempts to model the dynamics of military combat was the classical work of F. W. Lanchester (1916). His analysis consisted of two sets of simultaneous differential equations to describe the attrition of



opposing forces:

$$\frac{dn}{dt} = -\alpha m(t)n(t) \quad (1) \quad \frac{dm}{dt} = -\beta n(t)m(t) \quad (2)$$

$$\frac{dn}{dt} = -\alpha m(t) \quad (3) \quad \frac{dm}{dt} = -\beta n(t) \quad (4)$$

where:

$m(t)$  and  $n(t)$  number of surviving  $m$  and  $n$  forces at time  $t$

$\alpha$  the constant rate at which a single  $m$  unit kills  $n$  units

$\beta$  the constant rate at which a single  $n$  unit kills  $m$  units

Equations (1) and (2) are known as Lanchester's linear law formulation which is descriptive of two tactical situations. The first is a duel where each combatant can bring his weapon to bear upon only one opponent. The second case is long range combat where each combatant can fire only into an area in which the enemy is known to be but without knowledge of his exact location. Equations (3) and (4) are known as Lanchester's square law formulation. This describes combat at close quarters where the combatants attack each other in such a way that each unit may take any enemy unit under fire and, having killed that enemy unit, shifts its fire to another enemy unit. Derivation of the time and state solution for both sets of equations is straightforward. The reader may refer to Kimball and Morse (1951, pp. 65-67) which presents the solutions and some interesting conclusions derived from them.

Since Lanchester's original formulation of the equations, many analysts-- in an attempt to add reality and specificity to the combat situation--have extended and organized the theory (Brown, 1955) (Brackney, 1956) (Bach, Dolansky,

Stubbs, 1962). Details of the many extensions and applications are summarized in Bonder (1963).

In Lanchester's equations,  $\alpha$  and  $\beta$  were defined to be constant kill rates. Bonder (1963) shows that the m-force kill rate  $\alpha$  is explicitly related to a number of performance factors shown in Figure 13 (detection and firing times, hit probabilities, etc.) and the ballistic protection capabilities of the enemy. Similar functional relationships apply to the n-force kill rate  $\beta$ . Factors such as hit probabilities, firing times, and others noted in Figure 13 are known to be highly dependent on the range between opposing forces. Accordingly,  $\alpha$  and  $\beta$  are functions of range and therefore are variables in a combat situation when either or both forces are moving. This is denoted as  $\alpha(R)$  and  $\beta(R)$  where  $R$  is the range between opposing  $m$  and  $n$  forces. Rewriting the Lanchester equations with this notation:

$$\frac{dn}{dt} = -\alpha(R)m(t)n(t) \quad (5) \quad \frac{dm}{dt} = -\beta(R)n(t)m(t) \quad (6)$$

and

$$\frac{dn}{dt} = -\alpha(R)m(t) \quad (7) \quad \frac{dm}{dt} = -\beta(R)n(t) \quad (8)$$

Let us now investigate the utilization of these equations to describe a number of dynamic combat engagements where either or both combatants are moving. Consider first the close quarters meeting engagement where both  $n$  and  $m$  forces move toward each other. The square law is applicable to this situation. Using the transformation  $t = f(R)$  to transform all the variables into functions of range, and realistically treating  $\alpha$  and  $\beta$  as functions of

range. the following differential equation can be derived to describe the number of surviving  $n$  forces as a function of range between combatants.

$$\frac{d^2n}{dR^2} + \left( -\frac{a}{v^2} - \frac{1}{\alpha} \frac{d\alpha}{dR} \right) \frac{dn}{dR} - \frac{\alpha\beta n}{v^2} = 0 \quad (9)$$

where:

$v$  = relative speed between forces

$a$  = relative acceleration between forces

An analogous equation for the number of surviving  $m$  forces can be obtained by interchanging  $m$  for  $n$  and  $\beta$  for  $\alpha$ . Although equation (9) is just a description of the combat dynamics which remains to be solved, a number of interesting inferences may be drawn from it. First, the description, and thus the solution, contains factors of mobility (acceleration and speed), firepower (in the  $\alpha$  kill rate) and protection (in the  $\beta$  kill rate) of the  $m$  force. Second, as noted by the coefficient of the  $dn/dR$  term, the solution is dependent on the rate of change of the  $\alpha$  kill rate. The solution for the remaining  $m$  forces is analogously dependent on the rate of change of the  $\beta$  kill rate. If we had considered  $\alpha$  and  $\beta$  constant over all ranges per equations (3) and (4), the change in kill rates with range obviously, and unrealistically, would not influence the solutions.

Consider next, the description of a single tank-to-tank duel. The linear form applies. Employing the transform  $t = f(R)$ , equations (5) and (6) are used to derive the following description for the number of surviving  $n$  forces:

$$\frac{d^2n}{dR^2} + \left[ \frac{a}{v^2} + \frac{\beta n}{v} - \frac{1}{\alpha} \frac{d\alpha}{dR} \right] \frac{dn}{dR} - \frac{1}{n} \left( \frac{dn}{dR} \right)^2 = 0 \quad (10)$$

Again, an analogous equation for the number of surviving  $m$  forces can be obtained by interchanging  $m$  for  $n$  and  $\beta$  for  $\alpha$ .

Finally, consider the engagement which is probably most common to actual combat--attack by one force and defense of a prepared position by the other. If we let  $m$  be the attacking force, equations (5) and (8) apply resulting in the following description for remaining  $m$  and  $n$  forces respectively.

$$\frac{d^2m}{dR^2} + \left[ \frac{a}{v^2} + \frac{\alpha m}{v} - \frac{1}{\beta} \frac{d\alpha}{dR} \right] \frac{dm}{dR} = 0 \quad (11)$$

$$\frac{d^2n}{dR^2} + \left[ \frac{a}{v^2} - \frac{1}{\alpha} \frac{d\alpha}{dR} \right] \frac{dn}{dR} - \frac{1}{n} \left( \frac{dn}{dR} \right)^2 - \alpha \beta n^2 = 0 \quad (12)$$

Both equations are presented to point out an interesting difference between the description of symmetric (both forces moving) engagements and that of non-symmetric (one force moves) engagement. In the latter case, the solutions for surviving  $m$  and  $n$  forces are both dependent on the rate of change of the attacker's kill rate. Since the attacker controls the rate that kill rates change with range (by changing attack speed), this phenomenon might offer an explanation for the advantages of an attack posture.

In summary, a generalization of the Lanchester attrition theory as a model for Phase II analysis is being investigated. The generalization to variable kill rates appears to be a logical method of realistically integrating

weapon characteristics and combat operations. It is a means of explicitly including the many factors that comprise mobility, firepower, and protection. Using the Phase I relationships (which include environmental parameters of soil characteristics, terrain geometry, etc.) engagements in different environments can be analyzed. A change in enemy weapons can be taken into account through the kill rate functions  $\beta$  and  $\alpha$ . Tactics can be varied by changing such factors as acceleration, speed, number of forces employed, and initial firing ranges.

Future research activity will be directed to inclusion of additional relevant factors in the description of combat situations and solution of these descriptions by analytic and numeric methods.

## COMPONENT INTERACTIONS

by

S. Bonder, E. C. Sambuco, and F. B. Cook

### Introduction

The component hardware interaction problem and its relation to the tank performance equations:

$$y_1 = f(x_1, \dots, x_n; \theta_1, \dots, \theta_p) \quad (1)$$

was described in the annual report (Howland, Bonder, 1963, pp. 164-166). At that time it was proposed that two-dimensional equations between the hardware variables:

$$x_i = g(x_k) \quad i \neq k \quad (2)$$

be used to provide the requisite interaction information. This decision was based on the existence of a number of two-dimensional equations of this type which were developed by earlier armor studies (Noville, 1956) (Hill, Smith, & Weiss, 1950). If, as qualitatively shown in equations (1), there are  $n$  hardware variables, and if we assume that only one-way interactions occur, the number of possible equations  $m = n(n-1)/2$  becomes much greater than the number of variables  $n$  when  $n > 3$ . To eliminate the inconsistencies and/or nonindependent equations generated by having more equations than unknowns, a feasibility study was conducted to develop a method of reducing the number of equations (Sambuco, 1963). The study, which evaluated measures of interaction strength in interaction matrices, became increasingly complex as the number of equations increased. To circumvent this complexity, an alternate solution is under development.

#### Method

Qualitatively, the modified procedure which is being developed to account for component interactions can be described as follows: The  $n$  hardware variables (brake horsepower, main gun muzzle velocity, gross weight, crew compartment volume, etc.) are being divided into two categories; a dependent set ( $x_1, \dots, x_r$ ) and an independent set ( $x_{r+1}, \dots, x_n$ ). Based on physical considerations and the hardware characteristic data presently being collected, the following system of  $r$  multidimensional equations in  $n$  unknowns are under development:

$$\begin{aligned}
 x_1 &= g_1(x_2, \dots, x_r, \dots, x_n) \\
 x_2 &= g_2(x_1, x_3, \dots, x_r, \dots, x_n) \\
 &\vdots \\
 x_r &= g_r(x_1, \dots, x_{r-1}, x_{r+1}, \dots, x_n)
 \end{aligned}
 \tag{3}$$

The  $(n-r)$  independent hardware variables are those the military planner may specify and are the performance characteristics of components he wants used in the next generation armored system. For example, engine BHP, armor distribution, and main gun caliber. By specifying numerical values for the independent variables, the system of equations (3) is reduced to  $r$  equations in  $r$  unknowns, and, therefore, unique values for the remaining  $r$  dependent variables such as crew compartment volume, gross weight, and vehicle width, can be determined. Numerical values for all hardware variables --  $x_1$  through  $x_n$  -- are then used as inputs to equation set (1) for calculation of system capabilities (military characteristics) such as first round hit probability, ballistic protection, time to detect targets, cross-country speed, and acceleration. These in turn are used as inputs to the operational model described on pages 47 to 53 of this report. If the level of combat performance predicted by the model is not satisfactory, a change in the value of one or more of the independent hardware variables is made and the computational procedure repeated.

As noted above, a number of equations in set (3) will be formulated empirically. Accordingly, a composite data collection form was developed and

is presently being reviewed for completeness by the Armor Agency. The data forms are used to record hardware characteristics of components used in the tank models listed below. The list was developed jointly with the Armor Board, the Armor Agency, and The Systems Research Group staff. It is considered representative of tank models possessing different degrees of firepower, mobility, and protection.

- |                             |                          |
|-----------------------------|--------------------------|
| 1. M60A1 Main Battle Tank   | 11. M10A1 Tank Destroyer |
| 2. M48A1 Main Battle Tank   | 12. T32 Medium           |
| 3. M48A1E1 Main Battle Tank | 13. M26 Medium           |
| 4. T95E2 Medium             | 14. M4A1 Medium          |
| 5. M46 Medium               | 15. M4A2 Medium          |
| 6. M45 Medium               | 16. M4A3E2 Medium        |
| 7. M36 Tank Destroyer       | 17. M4A3E8 Medium        |
| 8. M36B1 Tank Destroyer     | 18. M3 Light             |
| 9. M36B2 Tank Destroyer     | 19. M3A1 Medium          |
| 10. M10 Tank Destroyer      | 20. M3A5 Medium          |

Research effort to date has been directed to a qualitative formulation of a solution to the interaction problem, the specification of a comprehensive set of hardware variables ( $x_i$ ), and development of a comprehensive form to be used in collection of hardware data for tank models noted above. Major activity in the forthcoming months will be data collection and generation of the set of interaction equations (3). It is anticipated that working visits to Detroit



Arsenal. Aberdeen Proving Grounds and the Armor Board will be required to obtain the necessary data. These agencies are presently being contacted by personnel of the Armor Agency.

## REFERENCES

- Bach, R. E., Jr., Dolansky, L., & Stubbs, H. L. Some Recent Contributions to the Lanchester Theory of Combat. Opns. Res. 10, 314-326, 1962.
- Bekker, M. G. Theory of Land Locomotion. University of Michigan Press. Ann Arbor. 1956.
- Bekker, M. G. Off the Road Locomotion. University of Michigan Press. Ann Arbor. 1960.
- Bonder, S. Exploratory Study of a Model for Phase II--Combat Effectiveness Analysis. Unpublished working paper RF-573-WP-54 (U). Systems Research Group, 1963. On file Ohio St. Univ., Columbus, Ohio.
- Boynton, R. M. & Bush, W. R. Recognition of Forms Against a Complex Background. J. Opt. Soc. of Amer., Vol. 49, No. 9, 758-764. 1956.
- Brackney, H. The Dynamics of Military Combat. Opns. Res. 7, 30-44, 1959.
- Brodkin, H. Fire Control Studies; Tank Gunnery Studies (U). Frankford Arsenal. Report No. R-1380, February 1957. (C)
- Brown, R. H. A Stochastic Analysis of Lanchester's Theory of Combat. The Johns Hopkins Univ. Operations Research Office. Technical Memorandum ORO-T-323. 1955.
- Bussman, D. R. Hard Soil Mobility. Unpublished working paper RF 573-WP-52 (U). Systems Research Group, 1963. On file Ohio State Univ., Columbus, Ohio.
- Eckles, A. J., Kibel, B. M., Littrell, R. M., Scriggins, T. D., & Agin, N. I. Effects of Trunnion Cant and Proposed Correction Devices (U). Research Analysis Corp., RAC-TP-86. February 1963 (S).
- FM 17-12 Tank Gunnery April 1961.
- Goldman, D. E. A Review of Subjective Responses to Vibratory Motion of the Human Body in the Frequency Range 1 to 70 Cys Project N.M. 004001. Naval Medical Research Inst., March 16, 1948.
- Hardison, D. C., Killian, R., Wolfe, I., & Fieldman, G. A Partial Analysis of Project Stalk Data with Results of Single Tank vs Single Tank Duels (C). Aberdeen, Maryland: Aberdeen Proving Ground. Ballistic Research Laboratories. February 1955.

- Hill. F. I. . Smith. R. & Weiss. H. K. An Introductory Study of the Characteristics of Tanks. Guns. and Armor (U). BRLM-505 (ATI-196 140). 1950 (C).
- Howland. D. The Trade-Off Problem in Weapon System Design. Military Review. XLIII. No. 10. October. 1963. 72-78.
- Howland. D. & Bonder. S. The Tank Weapon System. Report No. RF 573 AR 63-1 (U). Systems Research Group. Ohio State Univ. . Columbus. Ohio. 1963.
- Lamar. E. S. Operational Background and Physical Consideration Relative to Visual Search Problems. (In) Visual Search Techniques. Morris. A. & Hoene. E. P. (Eds.), NRC Committee on Vision. National Academy of Sciences. Washington. D. C. . 1960.
- Lanchester. F. W. Aircraft in Warfare; the Dawn of the Fourth Arm. London: Constable & Co.. 1916.
- Mackworth. N. H. Some Suggested Uses for the Optiscan--A Head Mounted Eye Camera. ASME Publication. paper Number 60-WA-304. The American Society of Mechanical Engineers (Eds.) New York. 1960.
- Mahig. J. Mobility on Rough Terrain. Chapter 2.2 in The Tank Weapon System. Report No. RF 573 AR 63-1 (U) Systems Research Group. Ohio State Univ. . Columbus. Ohio. 1963.
- Moore. J. M. Plant Layout and Design. New York: Macmillan Company. 1962.
- Morse. P. M & Kimball. G. E. Methods of Operations Research. New York: John Wiley & Sons. Inc. . 1951.
- Noville. G. O. & Assoc. Tank Design Parameters (U) Vol. II. Books I & II. AD 147 128 & AD 147 129. 1956. (S).
- Perloff. W. H. & Nair. K. Mobility in Soft Soil Chapter 2.1 in The Tank Weapon System. Report No. RF 573 AR 63-1 (U) Systems Research Group. Ohio State Univ. . Columbus. Ohio 1963.
- Sambuco. E. C. Interaction Theory. Unpublished working paper RF-573-WP-53 (U). Systems Research Group. 1963. On file. Ohio State Univ. . Columbus. Ohio.

Stollmack. S. Detection Probability. Chapter 3.3 in The Tank Weapon System. Report No. RF 573 AR 63-1 (U) Systems Research Group. Ohio State Univ., Columbus. Ohio. 1963.

Stollmack. S. Target Acquisition. Unpublished working paper RF-573-WP-50 (U). Systems Research Group. 1963. On file Ohio State Univ., Columbus. Ohio.

Williams. R. L. Interaction of Cant and Parallax. Chapter 3.2 in The Tank Weapon System. Report No. RF 573 AR 63-1 (U) Systems Research Group. Ohio State Univ., Columbus. Ohio. 1963.

## PUBLICATIONS AND PAPERS

Howland, D. The Trade-Off Problem in Weapon System Design. Military Review, XLIII, No. 10, October 1963, 72-78.

Perloff, W. H., Jr., & Nair, K. A New Approach to Off-the-Road Mobility of Tracked Vehicles on Soft Soil. Preprint of a paper presented at the Second Canadian-American Regional Meeting of the International Society of Terrain-Vehicle Systems, September 1963.

## RF 573 UNPUBLISHED WORKING PAPERS

1 July 1963 - 31 December 1963

No.	Date	Title	Author
50	17 Sept '63	Target Acquisition (U)	S. Stollmack
51	19 Nov '63	Thesis Project: "Analysis of the Relation of Tank Component Performance to Task Performances in the Tank Versus Tank Task." (U)	W. Bercaw
52	5 Dec '63	Hard Soil Mobility Progress Report (U)	D. R. Bussman
53	10 Dec '63	Interaction Theory (U)	E. Sambuco
54	17 Dec '63	Exploratory Study of a Model for Phase II-- Combat Effectiveness Analysis (U)	S. Bonder

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